PHYSICAL REVIEW B VOLUME 2, NUMBER 6 15 SEPTEMBER 1970

Nonresonant Interband Faraday Rotation in Silicon

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(Received 12 March 1970)

Faraday rotation has been measured in intrinsic silicon with a magnetic field of 20 kG at room temperature in the spectral region 0.45–2.05 eV. The measurements are discussed in terms of the theories of Kolodziejczak, Lax, and Nishina (KLN) and Boswarva, Howard, and Lidiard (BHL). Interpretation of the measurements according to these theories indicates that the major contribution to the Faraday rotation in this region is from direct transitions, and only a small contribution from indirect transitions is present. From the theory of KLN, a direct energy gap of 3.07 eV and an indirect energy gap of 1.18 eV were obtained. From the theory of BHL, a direct energy gap of 2.80 eV and an indirect energy gap of 1.16 eV were obtained. The two theories could not be distinguished on the basis of dispersion in the region of these measurements.

I. INTRODUCTION

The Faraday effect in n-type silicon in the energy region below the optical absorption edge has been investigated by several workers. Ukhanov[1] concerned himself mainly with the free-carrier Faraday effect and Stramska *et al.*[2] have not yet published an interpretation of their measurements. Piller and Potter,[3] mainly interested in the low-energy behavior of interband transitions, have interpreted their measurements in terms of indirect transitions. The energy squared dependence which they obtained right up to the indirect gap suggests that other types of transitions also contribute.[4]

In order to investigate the contribution of these other transitions, we have made measurements of the Faraday rotation in n-type silicon at room temperature in a magnetic field of 20 kG within the photon energy region 0.45–2.05 eV using apparatus designed specially for thin highly absorbing samples. Similar studies of interband transitions have been made in some II-VI compounds by Ebina *et al.*,[5] and Balkanski *et al.*,[6] and in ZnO by Baer.[7]

As in the above studies of interband Faraday rotation, we shall interpret our measurements in terms of the theoretical models of Kolodziejczak, Lax, and Nishina,[8] referred to here as KLN, and Boswarva, Howard, and Lidiard,[9] referred to here as BHL. These two theoretical treatments give expressions for the Faraday rotation caused by direct allowed transitions, direct forbidden transitions, and indirect phonon-assisted transitions. These theories are appropriate for simple nondegenerate isotropic parabolic conduction and valence bands. The frequency dependences obtained for each type of transition are similar for the two theories; however, they yield somewhat different energy gaps. The energy-band structure of silicon is more complicated than the simple model upon which these theories are based.[10–12] However, an interpretation according to these theories should still give an indication as to the types of transitions involved and their associated transition energies.

II. EXPERIMENTAL APPARATUS

The slab-shaped samples are placed within the gap of an iron-core electromagnet so that the magnetic field lies along the normal to the sample surface. A plane-polarized monochromatic light beam chopped by a mechanical disk chopper enters the magnet gap radially and is directed along the magnet axis by means of a mirror placed in the gap. The beam passes through the sample normal to its surface and is removed radially from the gap by a second mirror within the gap. The light beam then passes through a Wollaston prism beam splitter, and the two resulting beams, plane polarized perpendicularly to each other, are focused upon separate photodetectors. The two signals are added and subtracted simultaneously be means of differential and summing preamplifiers. The sum and difference signals are further amplified and detected by means of two phase-sensitive amplifiers. The reference signal being obtained from a solar cell and incadescent light bulb attached to the light chopper. Because the signal-to-noise ratio is generally poor for measurements made in the absorbing regions of the sample, it is desirable to integrate the signals for periods of the order of minutes. However, to obtain the Faraday rotation, the direction of the magnetic field is reversed several times during the measurement, making it desirable that the apparatus have a response time of the order of seconds. Therefore, the voltage signals from the phase-sensitive detectors are converted to frequencies and then counted for a fixed but arbitrary gate period. This arrangement gives an integration time equal to the gate period while the response time is determined by the response times of the amplifiers.

In order to cover the spectral region, two versions of the apparatus were used. The first, used for lower photon energies, utilized a 25-W tungsten lamp as a source, an $f/7$ Leiss prism single monochromator equipped with a CaF_2 prism and operated with a resolving power of about 20, and lead sulfide photodetectors. The radiation was chopped at 510 Hz. The second, used for the higher photon energies, utilized a 1600-W high-pressure xenon arc lamp as source, an $f/6$ Bausch and Lomb grating monochromator operated with a resolving power of about 140, suitable filters to eliminate higher orders, and silicon diode photodetectors. The radiation was chopped at 810 Hz.

The Glan Thompson polarizer was mounted in a smoothly operating rotator which could be driven by hand through a 1000:1 reduction gear train. A mechanical counter was used to keep account of the rotation. The least count of the system was 2×10^{-5} revolution.

III. MEASUREMENT TECHNIQUE

From the ratio ξ of the difference-to-sum signals, the Faraday rotation produced by the sample γ is obtained through the following relationship:

$$\tan 2\gamma = 2[\xi(-) - \xi(+)] \Big/ \left(\frac{d\xi(+)}{d\alpha} + \frac{d\xi(-)}{d\alpha}\right) \qquad (1)$$

in which the + and − arguments of ξ refer to measurements made with the magnetic field in forward and reversed directions and α is the angle of orientation of the polarizer measured with respect to the fixed analyzer. Equation (1) follows from the expression[13]

$$\xi(\pm) = g\rho \cos 2(\alpha \mp \gamma), \qquad (2)$$

where g is the ratio of the voltage gains in the difference and sum channels, respectively, and

$$\rho = [I(\max) - I(\min)]/[I(\max) + I(\min)]$$

in which $I(\max)$ and $I(\min)$ are, respectively, the maximum and minimum intensities obtained if an analyzer were placed in the light beam following the magnet and rotated through 180°. For values of γ less than 5°, Eq. (1) reduces to

$$2\gamma = \xi(-) - \xi(+) \Big/ \left(\frac{d\xi(\pm)}{d\alpha}\right)^{-1} . \qquad (3)$$

According to Eq. (1), the value obtained for γ is independent of ρ, α, and the source intensity; however, the sensitivity with which the measurement can be made increases as ρ approaches one, α approaches 45°, and the source intensity increases. A further requirement for Eq. (1) to apply is that the two photodetector systems must be matched to the degree that no change in the sum signal is observed as α is changed by an amount of several times γ. This requires, in addition, that the output beam of the monochromator be unpolarized.

In the measurements reported here, γ was always less than 1° so that Eq. (3) was used. The quantity $\xi(-) - \xi(+)$ was determined by averaging measurements made for successive reversals of the magnetic field, and $d\xi(\pm)/d\alpha$ was determined by rotating the polarizer in increments of the order of 10 min and obtaining the slope of the straight line of either $\xi(+)$ or $\xi(-)$ versus α by graphical or least-squares methods. At least five values of α were used in these determinations.

IV. DATA

Figure 1 shows the values of γ obtained for four n-type Si samples. Included in the figure are smoothed data points at 0.05-eV intervals obtained by a process of formation of normal places. That is, a least-squares fit to a second-degree polynomial was made in intervals of a few tenths of an electron volt for each sample, and the values of γ expected at energies separated by 0.05

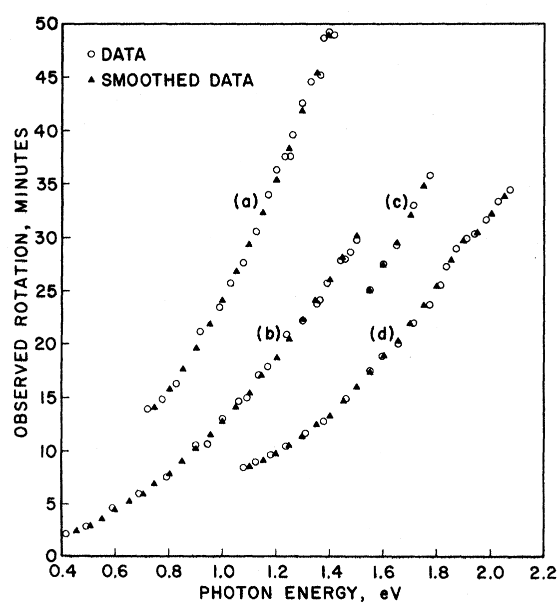

FIG. 1. Observed Faraday rotation in Si at 300 K for sample thicknesses and magnetic field strengths, respectively, of (a) 89.1 μm, 20.1 kG; (b) 46.5 μm, 20.2 kG; (c) 35.3 μm, 20.0 kG; (d) 24.2 μm, 20.0 kG.

eV were computed from these fittings. This process was carried out so that the data from different samples could be more easily combined and the resulting data handled more easily in further reduction. The positive sign of the rotation indicates that the rotation along the direction of propagation is in the same sense as the electric current in the magnet windings which produce the magnetic field.

The sample thickness was determined in two ways. First, by direct measurements of the edge thickness at several places using a traveling microscope. Second, by determining, according to a least-square method, the proportionality constants relating the measurements on each sample in regions of high transmission and of high absorption where the observed rotation is directly proportional to the thickness and then comparing these results to the measurements of Piller and Potter.[3] The measurements of Piller and Potter were used for normalization because they used a sample of sufficient thickness (2.05 mm) to make an accurate direct measurement. The thicknesses 89.1 μm, sample (a); 46.5 μm, sample (b); 35.3 μm, sample (c); 24.2 μm, sample (d) used are those obtained by the second method and are in agreement with those obtained by direct measurement.

The samples were single crystals. Samples (a) and (b) were intrinsic with 30–60-Ω cm resistivity, and samples (c) and (d) were possibly compensated with 1-Ω cm resistivity. Samples (a) and (b) were oriented with a $\langle 111\rangle$ direction normal to the sample surface, and samples (c) and (d) were oriented with a $\langle 111\rangle$ direction about 20° away from the normal to the sample surface.

In order to obtain accurately the Verdet coefficient from the observed rotation, two corrections must be considered: first, a correction to account for the multiple reflections and surface magneto rotation in the sample; second, a correction to account for the effect of the reflections occurring away from normal incidence on the two mirrors used to bring the radiation beam parallel to the applied magnetic field.

The effect of multiple reflections were compensated for by multiplying the observed rotation in each sample by the factor

$$(1-R^2e^{-2\eta d})/(1+R^2e^{-2\eta d}), \qquad (4)$$

where R is the reflection coefficient at normal incidence, η is the absorption coefficient, and d is the sample thickness. The derivation of this factor is given by Piller,[14] and Gabriel and Piller.[13] It applies to plane-parallel samples when the coherence length of the normally incident radiation is much less than the optical thickness of the sample, the rotation γ is less than 5°, and $k/n < 0.1$, where n and k are, respectively, the real and imaginary parts of the complex index of the refraction of the sample. The samples were not perfectly plane parallel; however, even if the samples are wedged to such an extent that only the first two reflected beams remain within the system, the correction factor for Si in the region of interest deviates from that given by Eq. (4) by less than $\frac{1}{2}\%$. The other conditions appear to be met. Because the optical thickness of the samples is many wavelengths, magneto-surface-rotation effects were neglected.[13]

An approximate calculation of the effect of the non-normal incidence on the mirrors was made assuming small Faraday rotation and taking into account the phase shifts and amplitude changes on reflection, but neglecting any magnetorotation at the mirror surfaces. Using the optical constants of Al as given by Heavens[15] and a 45° angle of incidence and a 45° angle of azimuth, which are appropriate to these measurements, we obtained a correction factor ranging between 0.99 and 0.98 in the energy region of interest. Since we are mainly interested in the energy dependence of the Faraday rotation, this factor was neglected. However, it should be pointed out that in the uv region this correction can be significant.

Values of R and η were obtained from the data published by Briggs,[16] Dash and Newman,[17] Salzberg and Villa,[18] and Philipp and Taft.[19] The data for n and η were formed into normal points in 0.05-eV increments in a manner similar to that used on the rotation data, and R was computed from n according to the relationship

$$R=(n-1)^2/(n+1)^2 . \qquad (5)$$

The values of R obtained in this way were in good agreement with the published values, so it was only a matter of computational convenience to use Eq. (5) to obtain R. A plot of the corrected and normalized quantity $n\gamma/Hd$, where H is the applied magnetic field, is shown in Fig. 2. The measurements reported here differ from those reported earlier[13] in that the earlier results showed a sharp decrease in rotation at about 1.6-eV photon energy. These more recent experiments indicate that this decrease is an instrumental effect which occurred when the signal-to-noise ratio of the sum channel became of the order of or less than 1. Measurements made with the same sample as used in Ref. 13, but using the second version of the apparatus show the onset of this decrease at a higher photon energy.

V. RESULTS

Each of the two theories we are considering gives expressions for the dispersion of the Fara-

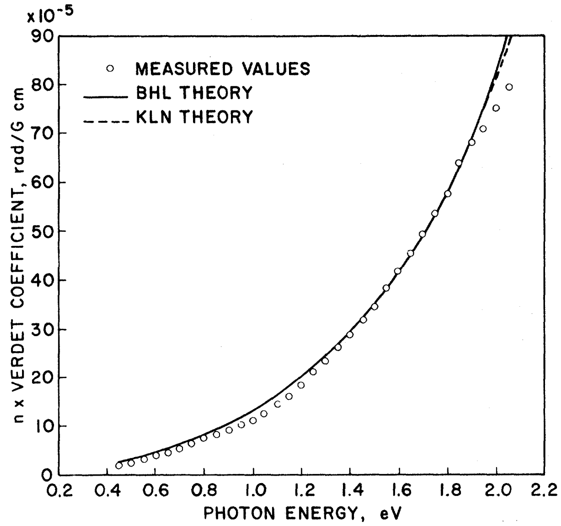

FIG. 2. The product of the Verdet coefficient and the index of refraction of Si at 300 K in a magnetic field of 20 kG. The curves are calculated from the theories of KLN and BHL for direct transitions with $E_g = 3.07$ eV, $K = 352.1\times10^{-5}$ rad/G cm and $E_g = 2.80$ eV, $K = 149.2\times10^{-5}$ rad/G cm, respectively.

day rotation as caused by allowed direct transitions, forbidden direct transitions, and indirect phonon-assisted transitions. All of these expressions are of the form $n\gamma/Hd = KF(E/E_g)$, where E is the photon energy, E_g is the energy gap appropriate to the transition, and K is a proportionality constant that is independent of E. For our present purpose, the form of K is unimportant. The expressions for $F(x)$, where $x = E/E_g$, according to the BHL theory are for the indicated transition: direct allowed transition,

$$F(x) = [(1-x)^{-1/2} - (1+x)^{-1/2}]\,x^{-1} - 1; \qquad (6)$$

direct forbidden transition,

$$F(x) = [(1+x)^{1/2} - (1-x)^{1/2}]\,x^{-1} - 1; \qquad (7)$$

indirect transition,

$$F(x) = 1 + \tfrac{1}{2}[(1-x)\ln(1-x) - (1+x)\ln(1+x)]x^{-1}. \qquad (8)$$

The corresponding expressions according to the KLN theory are for the indicated transition: direct allowed transition,

$$F(x) = [(1-x)^{-1/2} - (1+x)^{-1/2}]\,x^{-1} - 4[2 - (1-x)^{1/2} - (1+x)^{1/2}]\,x^{-2}; \qquad (9)$$

direct forbidden transition,

$$F(x) = [(1-x)^{1/2} - (1+x)^{1/2}]x^{-1} - \tfrac{4}{3}[2 - (1-x)^{3/2} - (1+x)^{3/2}]\,x^{-2}; \qquad (10)$$

indirect transition,

$$F(x) = 1 - [(1-x)\ln(1-x) + (1+x)\ln(1+x)]x^{-2}. \qquad (11)$$

These expressions hold in the region of $E < E_g$.

First an attempt was made to fit the data over the full energy range to a single type of transition by least-squares methods. However, it was found that these least-squares fits were only good in the region from 1.2 to 1.9 eV. In the region below the optical absorption edge at about 1.1 eV, one would expect to find a contribution to the rotation from indirect transitions because of the dominance of the indirect transition $\Gamma_{25'}$-Δ_1(min) in the absorption process. Above 1.9 eV, the deviation might also be caused by the contribution of other indirect transitions, for example, the $\Gamma_{25'}$-L_1 indirect transition which has been calculated to have an energy in the range 1.8–2.2 eV[12,20]; however, in this region the absorption coefficient is greater than 5×10^3 cm^{-1} so that even with the 24.2-μm-thick sample, the signal-to-noise ratio deteriorated to such an extent that integration times of the order of minutes were necessary, and possibly the deviation is due to the instrumental effect mentioned earlier.

The approach adopted was to ignore the data above 1.9 eV and make a least-squares fit to the data between 1.3 and 1.9 eV with a single type of transition. Then using the difference between the values calculated from the expression so obtained and the data in the region below 1.1 eV, make a least-squares fit to the expression for an indirect transition. The results of the least-squares fit between 1.3 and 1.9 eV are given in Table I. The values obtained for K, E_g, and the standard deviation of the fit σ defined as

$$\sigma^2 = \sum_{i=1}^{S} \frac{r_i^2}{S-2}, \qquad (12)$$

TABLE I. Results from the data in the region 1.3–1.9 eV.

Type of transition	E_g (eV)	K (10^{-5} rad/G cm)	σ(10^{-5} rad/G cm)	Theory
Direct allowed	2.80	149.2	0.450	BHL
Direct forbidden	2.28	511.9	0.552	BHL
Indirect	2.04	316.5	0.681	BHL
Direct allowed	3.07	352.1	0.408	KLN
Direct forbidden	2.47	−1173	0.479	KLN
Indirect	2.19	−355.5	0.563	KLN

TABLE II. Results from difference data in the region 0.45–1.05 eV.

Background transition	E_g (eV)	K (10^{-5} rad/ G cm)	σ (10^{-5} rad/ G cm)	Theory
Direct allowed	1.16	10.42	0.0761	BHL
Direct forbidden	1.36	18.28	0.0812	BHL
Indirect	1.71	35.06	0.0836	BHL
Direct allowed	1.18	−9.212	0.0737	KLN
Direct forbidden	1.33	−14.71	0.0794	KLN
Indirect	1.52	−23.15	0.0807	KLN

where s is the number of data points and the r_i are the residuals, are tabulated for each type of transition. In Table II, the values of K, E_g, and σ obtained from the least-squares fit of the difference data in the region 0.45–1.05 eV to the expression for an indirect transition are tabulated for each type of background transition.

From the values of σ given in Table I, one can infer that it is most likely that direct allowed transitions account for most of the Faraday rotation in the region 1.3–1.9 eV. The theory of KLN gives a somewhat better fit than that of BHL. Possibly because BHL have neglected the magnetic field dependence of the velocity matrix elements.[21] The values of σ given in Table II also tend to confirm that the background is caused by direct transitions. Furthermore, the values of 1.16 and 1.18 eV obtained from the theories of BHL and KHL, respectively, for the indirect energy gap by assuming a direct transition background rotation are in agreement with the value of the optical energy gap of 1.16 eV obtained from absorption measurements at 300 K.[22]

The curves shown in Fig. 2 are plots of the calculated values of $KF(E/E_g)$ for the values of E_g of 2.80 and 3.07 eV obtained for the direct transition energy from the theories of BHL and KLN, respectively. From these curves, it is apparent that the two theories cannot be readily distinguished on the basis of dispersion over the narrow photon-energy range provided by these measurements. Figure 3 shows the difference between these calculated values and the measured values as well as the curves calculated for indirect transitions with the energy gaps of 1.16 and 1.18 eV obtained from the theories of BHL and KLN, respectively. The steps in the data at 0.7 and 1.1 eV are produced by slight differences between the measurements on different samples.

Since the values obtained for the indirect energy gaps are in good agreement with the optical absorption edge leading one to attribute the indirect transition contribution to the $\Gamma_{25'} - \Delta_1(\min)$ transition, one is encouraged to speculate as to the identity of the dominate direct transition even though the theories considered here are not expected to apply to Si except possibly quite near the transition energy. The energy of the $\Gamma_{25'} - \Gamma_{15}$ transition from the valence-band maximum has been predicted by some authors[10,20] to be in the range 2.43–3.05 eV. Herman *et al.*[20] consider their best predicted value to be 2.8 eV for this transition. However, an energy of 3.5 eV for this transition has been inferred from the interpretation of absorption, reflectance, and photoemission measurements.[12] Furthermore, direct transitions at other points in the Brillouin zone such as $L_{3'}$-L_1 or Λ_3-Λ_1 transitions have predicted energies near 3 eV.[10,12,20] Thus, it does not seem possible to attribute the observed Faraday rotation to a particular transition on the basis of the theories of KLN and BHL. An extention of the measurements to energies greater than 3 eV might alter this situation. For this purpose, samples less than 10 μm thick would be required.

VI. CONCLUSIONS

We have found that if we assume that the Faraday rotation of Si at room temperature in the region of photon energy 0.45–1.90 eV contains a small contribution of opposite sign arising from an indirect transition superimposed additively on

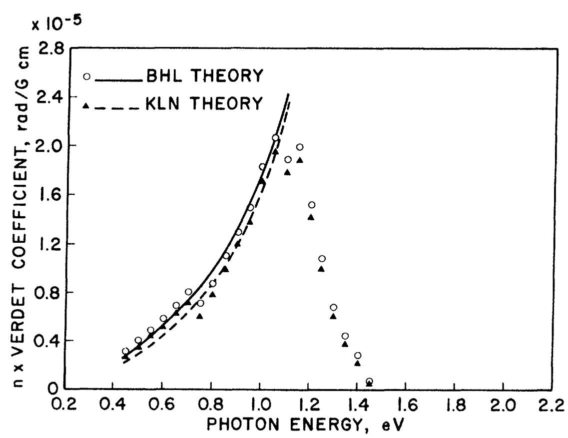

FIG. 3. The contribution from indirect transitions to the product of the index of refraction and the Verdet coefficient of Si at 300 K in a magnetic field of 20 kG. The curves are calculated from the theories of KLN and BHL with $E_g = 1.18$ eV, $K = 9.212\times10^{-5}$ rad/G cm and $E_g = 1.16$ eV, $K = 10.42\times10^{-5}$ rad/G cm, respectively.

a background rotation caused by a single type of transition, then the best least-squares fit of the theories of KLN and BHL to the data is obtained if the background is attributed to a direct transition. This choice is also supported by the agreement of the known value of the indirect optical energy gap with the indirect energy gap obtained from the least-squares fit. The theory of KLN yields a somewhat better least-squares fit than the theory of BHL. However, to distinguish between these two theories, it would be necessary to have data at photon energies closer to the energy gap. The direct energy gaps we have obtained from this interpretation of our data are significantly lower than $\Gamma_{25'}-\Gamma_{15}$ transition energy given by the interpretation of absorption, reflectance, and photoemission studies.[12] This may be a result of the simplified band structure assumed in the two theories we have used, or may simply indicate that further measurements should be made in the photon-energy region approaching the direct transition gap where theories that consider mainly the contribution of singularities in the rotation dispersion would be more applicable. Our results are in better agreement, however, with some predicted values for the $\Gamma_{25'}-\Gamma_{15}$ transition energy. Perhaps transitions at other points in the Brillouin zone contribute to the Faraday rotation so that the observed Faraday rotation can not be attributed to a particular transition.

ACKNOWLEDGMENTS

The author is indebted to Dr. H. Piller for many helpful discussions concerning this work and to J. Jerome for preparation of the samples.

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